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Tunable rheological behaviour of magnetized complex plasma

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ABSTRACT

The shear viscosity (η) of a 3D liquid dusty plasma has been estimated as a function of magnetic field (B) and normalized ion flow velocity (M) from the simulation data using Green–Kubo formalism with the help of Langevin dynamics simulation. It has been shown that in the strongly correlated liquid state, complex plasma may exhibit sharp changes in viscosity with magnetic field and ion drift velocity. In presence of ion drift, an oscillatory and attractive wake potential develops among charged dust particles immersed in a plasma. The amplitude of this wake potential can be modulated by applying an external magnetic field. In this work, we explore how an external magnetic field influences the rheological properties of complex plasma via anisotropic wake potential. It is observed that the rheological property of such plasma depends on the dominant interaction operating among the particles and can be controlled by applying an external magnetic field. A novel regime of magnetic field is observed in which strongly correlated complex plasma liquid exhibits a sharp response to an external magnetic field. Due to this unique property, complex plasma may be used as a platform to study magneto-rheological characteristics of soft matter and there is a possibility of using dusty plasma as a magneto-rheological material in near future.

1. Introduction

A complex plasma is an ensemble of charged dust particles immersed in a plasma medium [1]. The thermodynamics of strongly coupled dusty or complex plasma is mainly governed by two parameters: Coulomb coupling parameter $\Gamma = \frac{Q_d^2}{4\pi\epsilon_0 r_{av} K_B T_d}$, and screening strength $\kappa = \frac{r_{av}}{\lambda_D}$ where, r_{av}, λ_D, T_d and Q_d are the average inter-particle distance, Debye screening length, dust temperature and dust charge respectively. For suitable range of values of these two parameters, complex plasma may behave as soft matter exhibiting macroscopic softness or elasticity and sensitivity to external conditions [2]. Due to the relatively large size and mass of dust particles, it is easier to observe and resolve structural properties [3–5], phase transitions [6–10], etc. in such plasma and it provides an opportunity to study the phenomena shown by soft matter in such medium.

The electric fields present in RF sheaths or positive column of DC discharge of dusty plasma chambers often lead to flow of ions in plasma. The drifting ions get focused downstream the dust particles levitated in the sheath and perturb the Debye sphere around the grains. The resonant interaction of dust with the collective dust ion modes gives rise to an attractive oscillatory wake potential [11]. The existence of such wake potential has been extensively studied theoretically and confirmed by several experimental and simulation works [11–25].

The effect of resonant interaction between dust particles and low-frequency electrostatic ion-cyclotron wave on the emergence of oscillatory wake potential was studied by Nambu et al. [11] They examined how the wake potential behaves in presence of an external magnetic field. They assumed that the dominant wave vector perpendicular to the external magnetic field associated with ion cyclotron wave is in the ion flow direction. The strength of the wake potential was found to increase with magnetic field which was applied perpendicular to ion beam. The results are valid for (Mach number $M > 1$ and $\frac{1}{fM^2} < 1$ where $f = \frac{\omega_{pi}^2}{\omega_{ci}^2}$) supersonic flow of ions with relatively weak magnetic field. The effect of $E \times B$ drift on three dimensional wake potential for supersonic flow of ions with $f >> 1$ was discussed by Nambu et al. [23] The amplitude of the wake potential as obtained by Nambu et al. [23] has been plotted across magnetic field in Fig. 1(d) which shows increase in the strength of wake potential with magnetic field for the parameter regime mentioned therein. An expression for dy-

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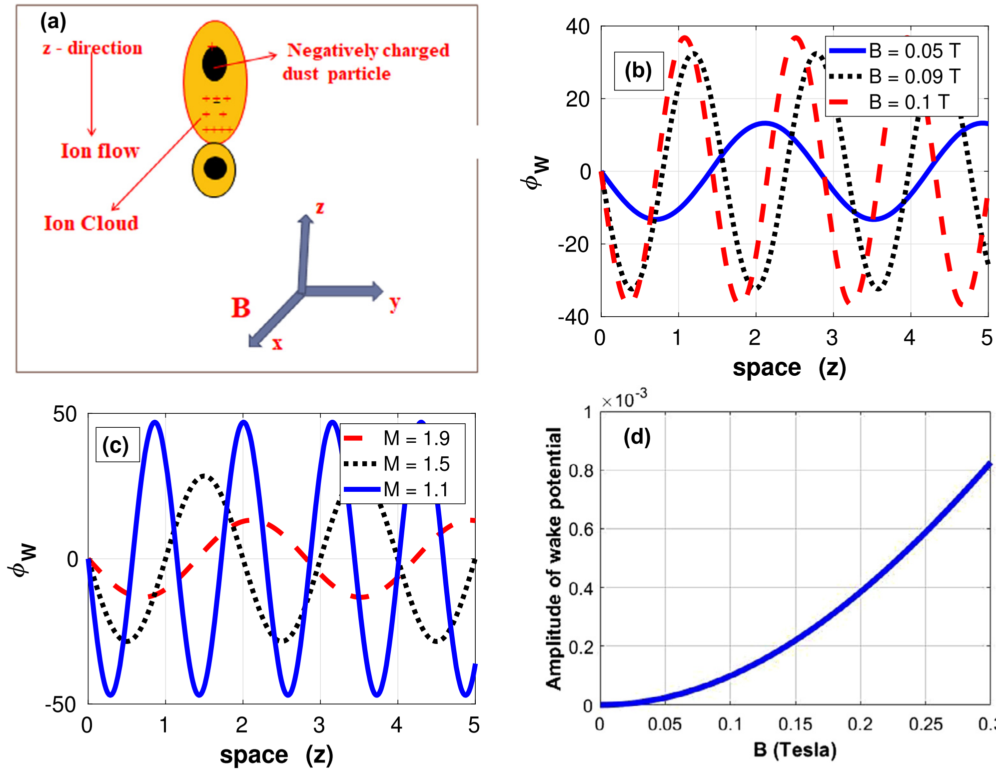

Fig. 1. (Colour online.) (a) Attraction between two dust particles in plasma with streaming ions and external magnetic field. (b) Shows the variation in the strength of wake potential (ϕ_W) for a set of magnetic fields for fix value of M = 1.9. (c) Shows the variation in the strength of wake potential (ϕ_W) for a set of Mach numbers when $B = 0.05$ T. (d) Variation of wake potential obtained by Nambu et al. [23] for z = 0, x = $\pi(M^2-1)^{\frac{1}{2}}\rho_e\ cos\alpha$, y = $\pi(M^2-1)^{\frac{1}{2}}\rho_e\ sin\alpha$, q_t = 10^{-16} C and M = 0.5.

namical potential was obtained by Nitta et al. [25] in the presence of magnetic field. They showed that in the subsonic/sonic regime, an oscillatory long range potential superimpose with the wake potential. The amplitudes of both the potentials are found to be proportional to the strength of the external magnetic field for $f > 1$. Bezbaruah et al. [13], on the other-hand obtained a simple expression for wake potential assuming that the wake is dominant along the ion flow direction in presence of a magnetic field applied perpendicular to the flow of ions. The theory is valid for magnetic field up to moderate to high value for supersonic ion flow. The expression for wake potential derived here is suitable for investing the effect of magnetic field and ion flow on dust crystal formation in line with the magnetized dusty plasma experiments currently running in various laboratories across the globe. Bhattacharjee et al. [26], also developed a theory on the formation of wake potential in presence of magnetic field both in subsonic and supersonic regimes. However, the theory had an upper-bound on the strength of magnetic field, thus limiting the range of applicability.

In presence of large electric fields, the particles may align themselves along the ion flow direction [27]. The tunable and anisotropic inter-grain interaction among the dust grains makes complex plasma an ideal candidate for soft matter such that its structural and transport properties can be suitably designed. In presence of magnetic field, the amplitude of wake potential is modulated and as a result, there is a possibility of controlling the rheological behaviour of complex plasma by tuning the external magnetic field, thus resembling the magneto-rheological properties of conventional soft matter.

Magneto-rheological (MR) fluids are the smart materials that exhibit a sharp response to change in magnetic fields and make transition from solid-like to liquid-like states [28]. From the microscopic point of view, MR fluids, in general, consist of soft-magnetic micron-sized particles dispersed in nonmagnetic fluids such as an aqueous carrier fluid, hydrocarbon, or silicone oil [28,29]. In the absence of an applied magnetic field most MR fluids flow freely, as a Newtonian fluid [30]. However, in the presence of an external magnetic field, they exhibit a phase change from a liquid-like to a solid-like state because of the formation of chain like structures due to the induced dipole moment between the dispersed magnetic particles [28]. As a result, with the change in the strength of the applied magnetic field, such fluids show enhanced and controllable rheological properties such as shear viscosity, dynamic modulus, yield stress, etc. [28–31]. Due to this tunability and reversible change in the mechanical characteristics, MR materials have received significant attention in both academic and industrial areas [28,29].

Shear viscosity is an essential transport property, which measures the internal resistance of a fluid. In the past few decades, transport properties of strongly coupled plasmas have been investigated in detail through theoretical and simulation techniques [32–46]. Using equilibrium molecular dynamics simulation, Hamaguchi et al. calculated the shear viscosity of strongly coupled Yukawa system [32]. Shear thinning behaviour of 2D Yukawa liquid has been shown by Donko et al. using non-equilibrium molecular dynamics (MD) simulation [47]. Finding a minimum value of viscosity with temperature (or coupling parameter) in many experimental and simulation works is a distinctive feature of complex plasma which is not found in most simple liquids [32,38,42]. The minimum of viscosity arises from the temperature dependence of kinetic and potential contributions to momentum transport [32,38]. An experiment was performed by Hartmann et al. to calculate the static and dynamic shear viscosity of a single-layer complex plasma [33]. In 2011, the static viscosity and the wavenumber-dependent viscosity were calculated from the microscopic shear in the random motion of particles [37]. Recently, Feng et al. have studied the shear viscosity of 2D liquid dusty plasmas under perpendicular magnetic fields using Langevin dynamics simulations [48]. It has been found that, when a magnetic field is applied, the shear viscosity of a 2D liquid dusty plasma is modified substantially. The different variational trends in the viscosity with the external magnetic fields can be explained on the basis of the kinetic and potential parts of the shear stress under external magnetic fields [48].

In most of these works, it is assumed that the particles interact with each other via Yukawa interaction potential, thus ignoring the effect due to ion flow. In the work by Feng et al. [48], the effect of magnetic field was studied by considering Lorentz force in equation of motion. Here, we emphasize that simulation with Yukawa potential does not fully explain the behaviour of shear viscosity of strongly coupled dusty plasma in laboratory in presence of ion flow. Due to the complex behaviour of plasma particles in presence of ion flow, an attractive wake potential may arise along the direction of flowing ions that further gets modulated in presence of magnetic field. In the present work, shear viscosity is calculated using Green Kubo relation by considering the fact that in presence of ion flow and magnetic field, the effective inter-particle potential is superposition of isotropic Yukawa and anisotropic attractive wake potential.

In this article, we explore the rheological property of complex plasma which shows a transition from ordered to disordered phase accompanied by change in viscosity when external magnetic field is applied. The combined effect of repulsive Yukawa and attractive, anisotropic wake potential on viscosity of complex plasma is investigated. Due to the inclusion of wake potential, we observe several novel features of rheological properties of complex plasma which was not seen in previous studies. The model description and simulation technique are presented in Section 2 and 3 respectively. Results are discussed in Section 4.

2. Description of the model

We consider a 3D dusty plasma consisting of electrons, ions, neutral particles and micron-sized grains. The ions are streaming in the vertical Z - direction perpendicular to an external magnetic field applied along the X - direction. In absence of streaming, the charged dust particles interact via repulsive screened Coulomb or Debye–Hückel potential defined as:

$$\phi_Y = \frac{Q_d}{4\pi\epsilon_0 r} exp\left(-\frac{r}{\lambda_D}\right) \tag{1}$$

Here, $\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{\sqrt{\lambda_{De}^2+\lambda_{Di}^2}}$ is the Debye screening length, where $\lambda_{De} = (\epsilon_0 K_B T_e/n_{e0}e^2)^{1/2}$ is electron Debye length and $\lambda_{Di} = (\epsilon_0 K_B T_i/n_{i0}e^2)^{1/2}$ is ion Debye length. n_{e0}, n_{i0}, T_e, T_i, Q_d are the electron density, ion density, electron temperature, ion temperature and dust charge respectively. When the ions from the bulk of the plasma stream towards the electrode, through the dust grains suspended in the sheath of RF plasma chambers, they experience a strong attractive force and get focused in the downstream of dust particles. Accumulation of positive charges distorts the isotropic Debye sphere. The nearby dust particles get attracted to this ion focus and results in a strong attractive force between the grains in the vertical direction. This oscillating and attractive wake potential may be described by using the concept of interaction between dust ion acoustic waves and dust particles. The total potential can be calculated by using dielectric response function. The trajectory of the flowing ions and focusing get further influenced when a magnetic field is applied perpendicular to the ion flow direction. It is found that the focusing of plasma ions is enhanced in such a situation and the amplitude of the wake potential is increased [11,13,23–26,49]. The effective interaction potential among the dust particles in such flowing magnetized plasma may be expressed as a superposition of spherically symmetric Debye Hückel and anisotropic wake potential [13].

$$\phi = \phi_Y + \phi_W \tag{2}$$

Where, ϕ_Y is the Yukawa potential (eq. (1)) and ϕ_W is the wake potential defined as [13]

$$\phi_W = \frac{-Q\rho}{\lambda_{De}\epsilon_0}\frac{sin(\sqrt{\alpha\rho}z)}{2(\rho+1)(\rho\alpha+1)} \tag{3}$$

Here, $\alpha = \frac{P}{2M^2}$, ρ = -1 + $\sqrt{1+\left(\frac{R}{P}\right)^2}$, $P = M^2 - f_i^2 - 1$ and $R = 2Mf_i$ are normalized parameters, $M = \frac{u_{io}}{\omega_{pi}\lambda_{De}}$ is the Mach number, $f_i = \frac{\omega_{ci}}{\omega_{pi}}$ is the normalized ion gyro frequency, u_{io} is the ion drift velocity, ω_{pi} is the ion plasma frequency and ω_{ci} is the ion cyclotron frequency. The nature of this wake potential and its dependence on Mach number (normalized ion flow velocity) and magnetic field B is shown in Figs. 1(b) and 1(c). It is clearly seen that the external magnetic field B, can be adjusted to tune the amplitude of the wake potential. This property of tunable interaction potential of complex plasma is explored in this work as a possibility of using complex plasma as magnetorheological soft matter. It is well known that the transport properties like self-diffusion and shear viscosity depend on inter-particle forces. It is shown here how magnetic field can be used to control the viscosity of complex plasma via tunable wake potential. The study may have application in variety of strongly coupled dusty plasma systems in astrophysical, laboratory and industries.

3. Langevin dynamics simulation

The spatial ordering and time evolution of many-particle systems can be studied with the help of Molecular Dynamics (MD) or Langevin Dynamics (LD) simulation. Here, simulation is performed using a 3 dimensional Langevin Dynamics code. The LD code has been thoroughly bench-marked against known past results. The diagnostics implemented in the LD code can explicitly reveal various properties like crystallization, phase behaviour and transport properties of strongly coupled dusty plasma. In the current study, we have performed the simulation for 1372 dust particles immersed in a plasma medium to mimic a 3D dusty plasma system. For the charged dust particles the Langevin differential equation is given by [50,51]

$$m\ddot{r_i}(t) = Q_d(\dot{r_i}(t)\times B) - \nabla\sum_{i\neq j}\phi_{i,j} - \nu m\dot{r_i}(t) + \zeta(t) \tag{4}$$

Here, the first term in the right-hand side is the usual Lorentz force due to the external magnetic field. The second term gives the pair potential. In our simulation, we have modelled our system considering a binary Yukawa inter-particle interaction (eq. (1)) together with particle-wake interaction (eq. (3)). The third term is the frictional drag force acting on moving dust particles. The expression for ν (dust-neutral collision frequency) used in our simulation is given by: $\nu = 1.4\times\frac{8\sqrt{2\pi}}{3}m_n n_n a^2\frac{v_n}{m_d}$ where m_n, n_n, a, v_n and m_d are mass of neutrals, neutral density, dust radius, thermal velocity of neutrals and mass of dust respectively. For the considered parameter regime $\nu \approx 1$ Hz corresponding to $n_n \sim 10^{21}$ m^{-3}, $m_n \sim 10^{-27}$ kg, $v_n \sim 10^2$ m/s and dust mass $m_d \sim 10^{-18}$ kg for sub micron sized dust grain. $\zeta(t)$ is the random force due to thermal fluctuations of the plasma particles. The expression for $\zeta_i(t)$ is given by $\zeta_i(t) = \sqrt{\frac{\beta^2}{\delta t}} * N(0,1)$. β is an undetermined coefficient and in equilibrium it is related to the drag coefficient through fluctuation dissipation theorem, δt is the simulation time step and N(0,1) represents the normal random variable with mean zero and variance 1. In order to have a clear idea on the transport of particles in a real experimental situation, the contribution of streaming ions and the magnetic field have been incorporated via the relevant interaction potential operative among dust grains. The physical quantities length, mass, time, velocity and energy are normalized by λ_D, m_d, $\sqrt{\frac{m_d \lambda_D^2}{K_B T_d}}$, $\sqrt{\frac{m_d}{K_B T_d}}$ and $K_B T_D$ respectively.

The system is taken to be a cubical box of size $L_x = L_y = L_z = 10^{-3}$ m (Volume, V = $L_x L_y L_Z$) consisting of 1372 (say N) particles. The code implements velocity varlet algorithm to solve Langevin equation of motion which in turn yields positions of particles at every time step [50]. The Berendsen thermostat has been used to re-scale the velocities to control fluctuations in kinetic energy and thereby the desired temperature is set for the system as it evolves into equilibrium [50,52]. Periodic boundary conditions are imposed along all directions to ensure the conservation of particles [50,51]. In a particular run, the basic steps followed in the simulation are: (i) canonical ensemble (NVT) run: In this step, three parameters of the system are fixed throughout the simulation: number of particles (N), volume (V) and temperature (T). This step helps to take the system to a thermal equilibrium at the required Γ by using the thermostat. (ii) micro-canonical ensemble (NVE) run: The micro canonical ensemble represents an isolated system. In the second step, the thermostat is removed to perform a NVE run to bring the system to the desired equilibrium. Data collection starts after this step.

The Green–Kubo relations give the exact mathematical expression for transport coefficients in terms of integrals of auto-correlation functions. In this formalism, first the stress auto-correlation function (SACF) is being calculated as

$$C_\eta(t) = \langle P_{xy}(t).P_{xy}(0)\rangle \tag{5}$$

The angular brackets represent the averaging over the entire ensemble of particles. $P_{xy}(t)$ is the off-diagonal element of the stress tensor defined by the formula

$$P_{xy}(t) = \sum_{i=1}^{N}\left[m v_{ix} v_{iy} + \frac{1}{2}\sum_{j\neq i}^{N} \frac{x_{ij} y_{ij}}{r_{ij}} \frac{\partial \Phi(r_{ij})}{\partial r_{ij}} \right] \tag{6}$$

where i and j are indices for different particles, N is the total number of particles, $r_{ij} = |r_i - r_j|$. From this expression, it is evident that the stress auto-correlation function is connected with the interaction potential operating among the dust particles. In absence of ion flow or magnetic field, $\Phi(r_{ij})$ represents symmetric Yukawa potential. In the present model, it is assumed that the dielectric response function gets affected by the magnetic field applied perpendicular to the streaming of ions, which results in an asymmetric potential as defined in equation (3). Thus, the viscosity gets affected by external magnetic field and ion flow via the effective anisotropic inter-particle potential.

The first term in eq. (6) is the kinetic contribution: $P_{xy}^{kin}(t) = \sum_{i=1}^{N}\left(m v_{ix} v_{iy}\right)$ and the second term is the potential contribution: $P_{xy}^{pot}(t) = \left(\frac{1}{2}\sum_{i=i}^{N}\sum_{j\neq i}^{N} \frac{x_{ij} y_{ij}}{r_{ij}} \frac{\partial\Phi(r_{ij})}{\partial r_{ij}}\right)$. Therefore we can write eq. (5) as

$$\begin{aligned} C_\eta(t) &= \langle P_{xy}^{kin}(t).P_{xy}^{kin}(0)\rangle + \langle P_{xy}^{pot}(t).P_{xy}^{pot}(0)\rangle + 2\langle P_{xy}^{kin}(t).P_{xy}^{pot}(0)\rangle \\ &= C_\eta^{KK}(t) + C_\eta^{PP}(t) + 2C_\eta^{KP}(t) \end{aligned} \tag{7}$$

Here, $C_\eta^{KK}(t)$ is the self-correlation of the kinetic part, $C_\eta^{PP}(t)$ is the self-correlation of the potential part and $C_\eta^{KP}(t)$ is the cross-correlation. Once the SACF is obtained, the static viscosity η is being calculated from the Green–Kubo relation as [32,48]

$$\eta = \frac{1}{V k_B T}\int_0^\infty C_\eta(t)\,dt \tag{8}$$

Here, V is the volume of the simulation box, k_B is the Boltzmann constant and T is a temperature. After splitting $C_\eta(t)$ into 3 parts, we get from eq. (8)

$$\eta_{kin} = \frac{1}{V k_B T}\int_0^\infty C_\eta^{KK}(t)\,dt \tag{9}$$

$$\eta_{pot} = \frac{1}{V k_B T}\int_0^\infty C_\eta^{PP}(t)\,dt \tag{10}$$

$$\eta_{cross} = \frac{2}{V k_B T}\int_0^\infty C_\eta^{KP}(t)\,dt \tag{11}$$

Here, η_{kin}, η_{pot} and η_{cross} are the kinetic, potential and cross parts of shear viscosity respectively. Therefore, $\eta = \eta_{kin} + \eta_{pot} + \eta_{cross}$. It is to be noted that in the present study, we have modelled our system considering a binary isotropic Yukawa inter-particle interaction together with anisotropic particle-wake interaction. Since the isotropy of the system breaks in the presence of anisotropic wake potential and external magnetic field, therefore, in our simulation, each of the three components i.e., $C_\eta^{xy}(t)$, $C_\eta^{yz}(t)$, $C_\eta^{zx}(t)$ has been separately calculated [50].

The structural properties of the system can be computed using radial distribution function $g(r)$ [50]. It is a measure of the structural correlations between particles that may organize in solid or fluid like state. For a 3D system, radial distribution function (RDF) is defined as [50,51]

$$g(r) = \frac{V}{N}\frac{N(r,dr)}{4\pi r^2 dr} \tag{12}$$

To measure the existence of long range order in the system, an important diagnostic tool is Lattice correlation factor (LCF) [50,51]. LCF can be computed from the position of the particles. It can be measured experimentally through X-ray scattering technique [50]. The local density of particles $\rho(\mathbf{r})$, at a point $\mathbf{r}$, can be expressed as [50]

$$\rho(\mathbf{r}) = \sum_{j=1}^{N}\delta(\mathbf{r} - \mathbf{r}_j(t)) \tag{13}$$

where $\mathbf{r}_j(t)$ denotes the position of particle j at time t and N is the total number of particles. The density can be spatially Fourier transformed and its Fourier transform is simply the Lattice Correlation Factor (LCF) [50].

$$\rho(\mathbf{k}) = \frac{1}{N}\sum exp(-i\mathbf{k}.\mathbf{r}_j) \tag{14}$$

For perfectly crystalline state (fully ordered), $|\rho(k)| \approx 1$. Mean-squared displacement (MSD) [50] is used as an another diagnostic tool to determine the phase behaviour, which is defined as

$$\mathrm{MSD}(t) = \langle|\mathbf{r}_i(t) - \mathbf{r}_i(0)|^2\rangle \tag{15}$$

Where, $\mathbf{r}_i(t)$ is the position of the i^{th} particle at time t and the angular bracket denotes an ensemble average. By calculating the particle positions at different time steps, the mean-squared displacement (MSD) can be measured from the simulation data.

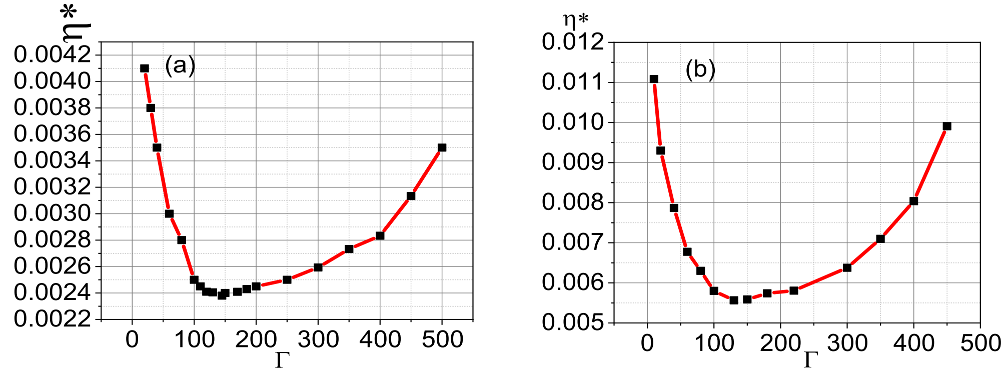

Fig. 2. (Colour online.) (a) Variation of normalized shear viscosity (η^*) with Coulomb coupling parameter (Γ) when B = 0.001 T, κ = 2 and ν = 1 Hz ($n_n = 10^{21}$ m^{-3}). The simulation was performed considering Yukawa potential only along with the Lorentz force term. (b) Normalized shear viscosity (η^*) versus Coulomb coupling parameter (Γ) considering both Yukawa and magnetized wake potential. All other simulation parameters are same as Fig. (a).

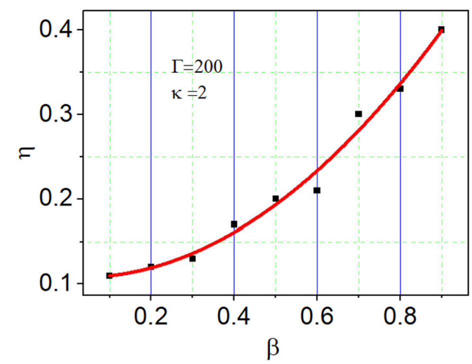

Fig. 3. (Colour online.) The variation of η with the dimensionless parameter $\beta = \frac{\omega_c}{\omega_{pd}}$ for 2D dusty plasma, where ω_c is the cyclotron frequency of the dust particle, and ω_{pd} is the nominal dusty plasma frequency. The external magnetic field is characterized using the dimensionless parameter β for M = 0, i.e., in the absence of anisotropic wake potential.

4. Results and discussions

We present the results of our simulation study of shear viscosity of three-dimensional strongly coupled complex plasma. The dependence of shear viscosity on magnetic field and ion flow velocity has been analyzed. The present work helps in realizing the explicit contribution of repulsive Debye–Hückel potential and attractive wake potential on the overall behaviour of viscosity in complex plasma.

(A) Variation of viscosity with Γ:

The variation of shear viscosity (η) with Γ is shown in Fig. 2(a). The simulation was performed with Yukawa potential only along with the Lorentz force term. A prominent feature of the η versus Γ curve is the minimum occurring at intermediate coupling $\Gamma \approx 150$. This minimum of η has been confirmed in many experimental and simulation works [32,38,42]. The minimum arises from the temperature dependence of kinetic and potential contributions to momentum transport. To understand this behaviour, we separate the stress tensor $P_{xy}(t)$ (eq. (6)) into two parts as: (i) the first term (kinetic part) $P_{xy}^{kin}(t) = \sum_{i=1}^{N}\left[m v_{ix} v_{iy}\right]$ and (ii) the second term (potential part) $P_{xy}^{pot}(t) = \left[\frac{1}{2}\sum_{i=i}^{N}\sum_{j\neq i}^{N}\frac{x_{ij}y_{ij}}{r_{ij}}\frac{\partial\Phi(r_{ij})}{\partial r_{ij}}\right]$. The kinetic part, depends on particle velocities and represents the momentum transport by the displacement of particles. The potential part depends on the effective pair potential between the dust particles. Therefore, the kinetic part of η (eq. (9)), decreases with Γ while the potential part of η increases with Γ. At high temperature (low Γ), the system behaves more like a gas. On the other hand, when Γ is larger than 150, η increases with Γ because the system gradually goes to a fluid regime and brings in regularity to the system and attains an ordered structure. Fig. 2(b) depicts the variation of shear viscosity (η) with Coulomb coupling parameter (Γ) considering both Yukawa and magnetized wake potential.

(B) Variation of viscosity with magnetic field:

In order to see the dependence of shear viscosity on magnetic field and compare it with the results of previous work reported by other authors, the simulation is performed with Yukawa potential and Lorentz force by putting M = 0 (absence of anisotropic wake potential). Fig. 3 represents the variation of normalized η with the dimensionless parameter $\beta = \frac{\omega_c}{\omega_{pd}}$, (where ω_c is the cyclotron frequency of the dust particle, and ω_{pd} is the nominal dusty plasma frequency) for Γ = 200 and κ = 2. In this parameter regime, the shear viscosity is found to increase with the external magnetic field. The results follow same trend as that of Feng et al. [48].

The primary objective of the present work is to explore the behaviour of shear viscosity in presence of magnetized wake. For this, the wake potential is introduced into the equation of motion. The variation of shear viscosity (η) with magnetic field is studied for the range of B from 0.001 T to 0.3 T as shown in Fig. 4(a). For the entire study, we have focussed in the supersonic regime of ion flow. The values of neutral density n_n, coupling parameter Γ, screening parameter κ and ion Mach number M are kept fixed at 10^{21} m^{-3}, 450, 2 and 1.9 respectively. The different variational trends of viscosity (η) with the external magnetic field can be explained on the basis of the behaviour of shear stress ($P_{xy}(t)$) under external magnetic fields. From Fig. 4(a), the entire range of observations can be classified into three regions.

Region I: This regime is characterized by ultra-low magnetic field (B = 0.001 T to 0.05 T), where the dust ensemble exhibits a sharp increase in the values of viscosity with magnetic field. The normalized value of viscosity shows a B^7 dependence in this regime. The plot of stress auto-correlation function (SACF) in Fig. 4(b) reveals that this function decays relatively weakly with smaller ripples for the range of magnetic field considered here. This novel behaviour of sharp response of viscosity to small change in magnetic field may be explained on the basis of tunable interparticle interaction of complex plasma. In this regime, the complex plasma behaves like a strongly correlated liquid with LCF lying between 0.42 - 0.232 as seen in Fig. 4(f). For the range of parameters chosen here, the Yukawa potential is dominant over wake potential (Fig. 4(e)). However due to the field-dependent attractive wake potential, the effective potential increases with magnetic field resulting in sharp change of viscosity with magnetic field. The potential part (($C_\eta^{PP}(t)$)) of SACF is found to dominate over the

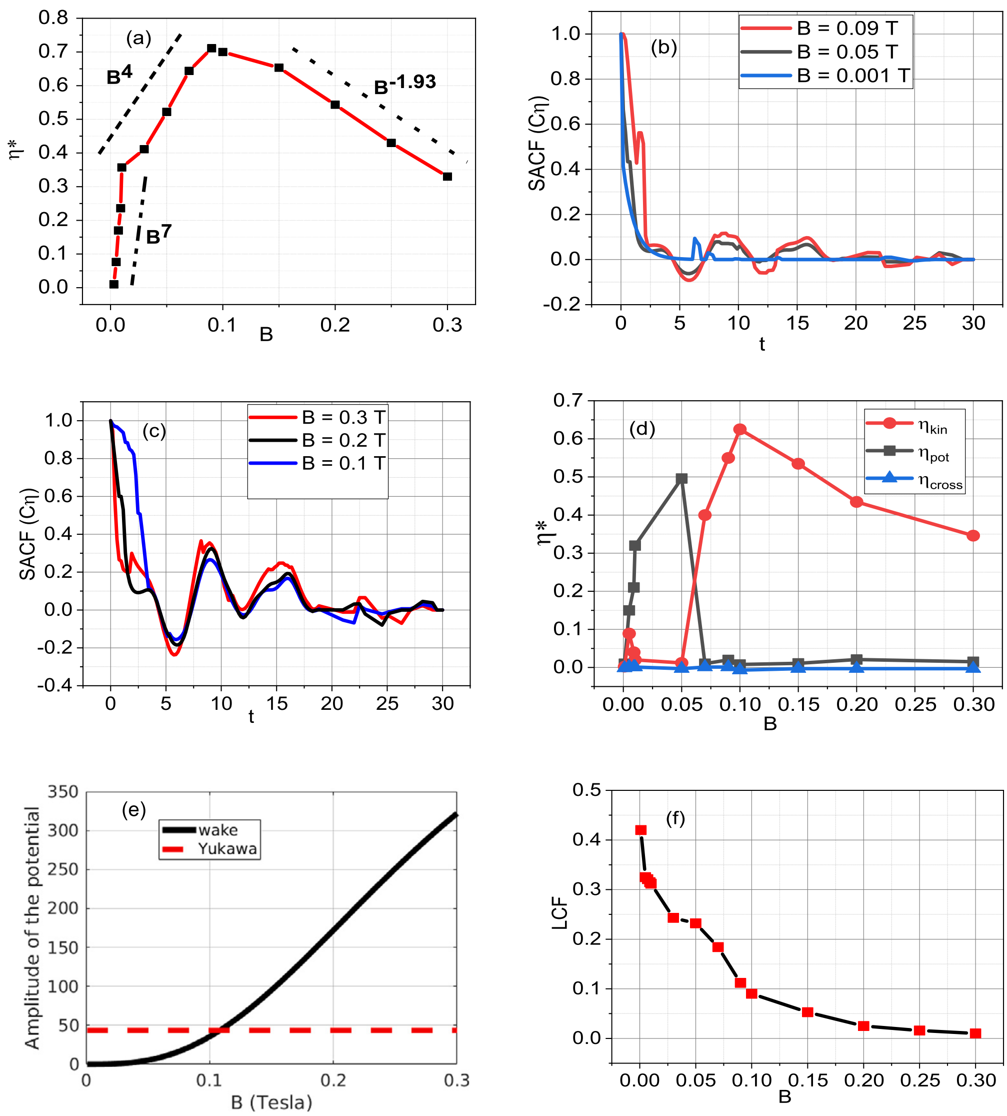

Fig. 4. (Colour online.) (a) Normalized shear viscosity ($\eta^{\star}$) versus magnetic field B (in Tesla) when $\Gamma = 450$, κ = 2, M = 1.9 and ν = 1 Hz ($n_n = 10^{21}$ m^{-3}). Figs. (b) and (c) - the stress auto-correlation function (SACF) for a range of B values. Fig. (d) - variation of kinetic, potential and cross part of viscosity with magnetic field. Fig. (e) - comparison of the amplitude of wake and Yukawa potential. Fig. (f) - Lattice Correlation Factor (LCF) for a range of B values.

kinetic part such that $\eta_{pot} > \eta_{kin}$ in this regime which may be attributed to dominant Yukawa interaction among the particles.

It is to be noted that the effective inter-particle potential is controlled by interplay between Yukawa and wake potential. Although Yukawa potential does not depend on the external magnetic field, the dependence of wake on magnetic field gets reflected in the behaviour of effective potential. For the range of magnetic field where the wake potential is weak, the transport property of the medium is decided mainly by Yukawa potential. The overall dynamics of the system is governed by the effective inter-particle potential. Although the kinetic part of η has no direct relation with magnetic field, it depends on particle velocity which is affected by the inter-particle interaction via Yukawa and magnetic field dependent wake potential.

Region II (B = 0.05 T to 0.09 T): The second regime is characterized by low magnetic field, continues to show a rise in viscosity with magnetic field with a lesser slope ($\propto B^4$). The Yukawa dominance of the effective potential gradually decreases although the plasma maintains strongly correlated fluid state, however with decreasing value of LCF. This observation is supported by the rise of the kinetic contribution of viscosity as seen in Fig. 4(d).

The trend of viscosity in this low magnetic field regime is in agreement with the results found in the work by Feng et al. in the cold plasma limit [48]. The shear viscosity was found to increase with external magnetic field. However, the sharp increase in η with B observed in our work in presence of supersonic ion flow was not visible in their work with isotropic Yukawa potential. We emphasize that the response of viscosity to external magnetic field can be made more efficient by suitably tuning the wake potential.

Region III (B = 0.09 T to 0.3 T): For relatively high magnetic field the viscosity exhibits a complete reversal with variation of magnetic field. Fig. 4(c) shows that the SACF decays rapidly with ripples having higher amplitudes. Here, the kinetic part of viscosity dominates over the potential part. It is found that η_{pot} drops to a low value in this regime, resulting in decreasing trend of viscosity with magnetic field. Here, we see the agreement of our result with the Braginskii equations [48,53], where the dependence of shear viscosity on magnetic field is estimated as $\eta \propto \frac{1}{B^2}$. The decreasing trend of η_{pot} and η with B may be attributed to the strong anisotropic wake potential operating among the dust particles in this regime. This can be seen from Fig. 4(e), where the effective

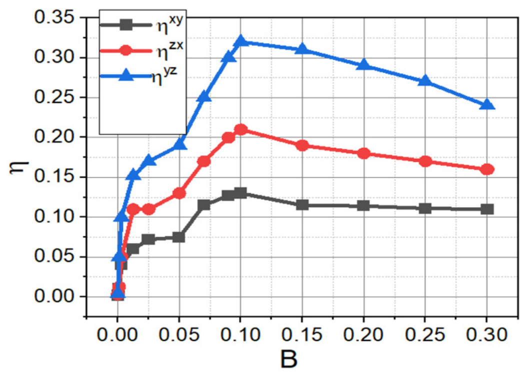

Fig. 5. (Colour online.) Variation of η^{xy}, η^{yz} and η^{zx} with the external magnetic field for M = 1.9, $\Gamma = 450$, κ = 2 and ν = 1 Hz ($n_n = 10^{21}$ m^{-3}).

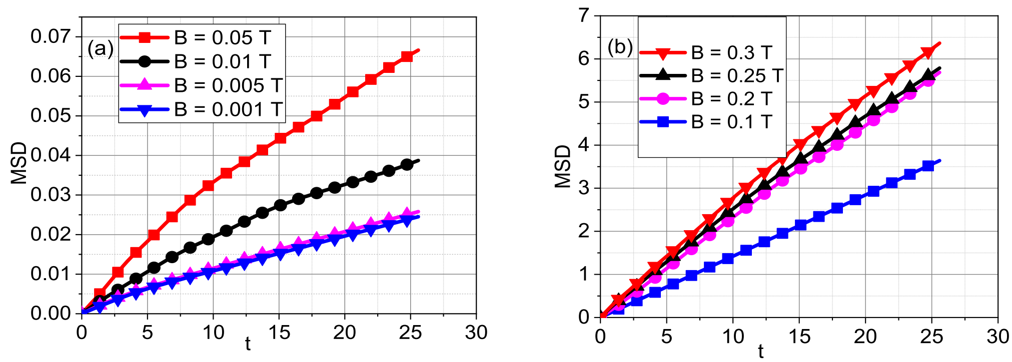

Fig. 6. (Colour online.) (a) and (b) - The evolution of mean-squared displacement (normalized value) for a range of B values when Γ = 450, $\kappa = 2$, M = 1.9 and $\nu = 1$ Hz ($n_n = 10^{21}$ m^{-3}). In the simulation, MSD and time are normalized by λ_D^2 and $\sqrt{\frac{m_d \lambda_D^2}{K_B T_d}}$ respectively.

potential becomes wake dominant and the complex plasma transits to gaseous state (LCF: 0.08 - 0.01).

The plot of variation of η^{xy}, η^{yz} and η^{zx} with external magnetic field for M = 1.9 as shown in Fig. 5 give interesting information about contribution of these components to shear viscosity (η). Here, the individual components are calculated separately based on the results of $p_{xy}(t)$ (as defied in equation (6)), $p_{yz}(t)$, $p_{zx}(t)$ respectively. In the regime of low magnetic field, the component η^{xy} exhibits very slow variation with magnetic field. The variation of η^{yz} is found to be very stringent in this regime of magnetic field and is due to the wake potential, operating in the Z-direction, perpendicular to the magnetic field. For relatively high magnetic field η^{yz} shows a decay with increase in magnetic field due to combined effect of Lorentz Force and wake potential. The component η^{xy}, almost remains steady with magnetic field beyond B = 0.15 T. Similarly, the dependence of η^{yz} and η^{zx} may be explained due to the combined effect of wake potential and many particle correlations in the strongly coupled liquid state.

In order to see the relation with other transport properties, we have also observed the evolution of mean squared displacement (MSD). From Figs. 6(a) and 6(b), it is explicit that the MSD increases with B. But the values of MSD from 0.001 T to 0.005 T (Fig. 6(a)) are much lesser than that of Fig. 6(b) (B from 0.1 T to 0.3 T). In this regime (B from 0.1 T to 0.3 T), the Lorentz force $Q_d\,(\dot{r_i}(t) \times B)$, starts to become effective and shows its influence on transport properties of the system. The MSD of the dust particles is also very high as depicted by Fig. 6(b). It is the combined role of Lorentz force and magnetized wake that is responsible for the decrease in the values of shear viscosity in this regime.

From above observations, we can infer that the phase state of the complex plasma is closely connected with the nature of interaction among the particles. For the range of magnetic field where repulsive and isotropic Yukawa potential is dominant such that the system is in strongly correlated fluid state, the potential part of viscosity increases with the magnetic field, resulting in sharp rise of viscosity. This is identified as a novel regime because the response of viscosity is very sensitive even for a small change in magnetic field. On the other-hand, in the regime, where the wake potential is dominant, due to its anisotropic character, the particles lose their strong correlation, tend towards a gaseous state thus exhibiting a decreasing trend of viscosity with rise in magnetic field.

(C) Variation of viscosity with ion flow velocity:

In order to understand the role of ion Mach number on shear viscosity, we perform simulation by varying Mach number M in the supersonic regime from 1.1 to 1.9. The results are shown in Fig. 7(a). In the range $1.1 < M < 1.5$, the shear viscosity does not show significant change. The plot of the amplitudes of Yukawa and wake potentials in Fig. 7(c) shows that this is the wake dominant regime, where the dusty plasma behaves like a gas. The LCF values (Fig. 7(d)) in this regime are less than 0.02 indicating the gaseous state. The MSD plot of Fig. 7(b) supports this observation.

The regime of interest starts from $M \geq 1.5$, when the wake potential becomes comparable to isotropic Yukawa potential. When the ions move faster, the accumulation of ions downstream the dust particles get reduced resulting in weakening of wake potential. The complex plasma attains a fluid like state due to the balance between repulsive Yukawa and attractive wake potential. In this strongly correlated fluid state, the shear viscosity increases with Mach number accompanied by decrease in MSD values. It may be concluded that dusty plasma exhibits high viscosity for moderate values of magnetic field in supersonic regime of ion drift.

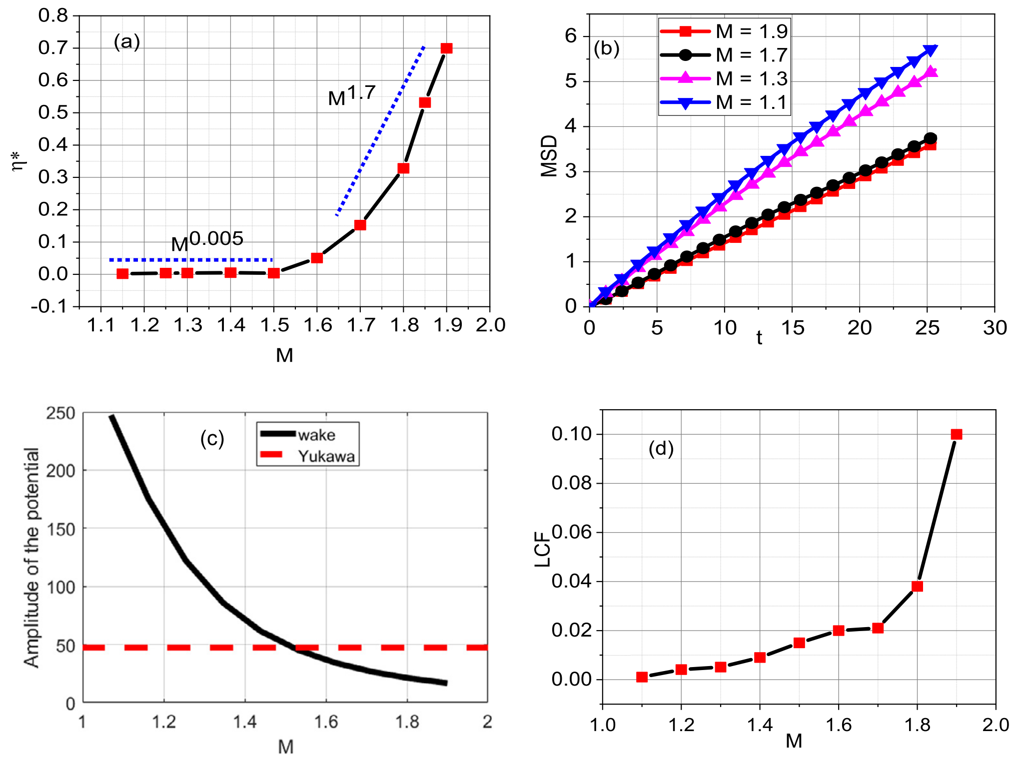

Fig. 7. (Colour online.) (a) Normalized shear viscosity (η^*) versus Mach number when $B = 0.1$ T, $\Gamma = 450$, $\kappa = 2$, and $n_n = 10^{21}$ m^{-3}. (b) The evolution of MSD for a range of M values when B = 0.1 T, Γ = 450, κ = 2, and n_n = 10^{21} m^{-3}. Fig. (c) - comparison of the amplitude of wake and Yukawa potential for a set of Mach numbers. Fig. (d) - Lattice Correlation Factor (LCF) for a range of M values.

5. Conclusion

The shear viscosity (η) of a 3D liquid dusty plasma has been estimated from the simulation data using the Green–Kubo formalism with the help of Langevin dynamics simulation. The dependence of shear viscosity on magnetic field and ion flow velocity has been analyzed. The novel feature of this work is that the viscosity of complex plasma is found to be sensitive even for small changes in magnetic field because of the role of tunable wake potential with magnetic field. The shear viscosity of dusty plasma has been extensively studied in recent years using analytical methods, simulation and experiments [32,33,37,38,42,47,54]. The reason for the discrepancy between simulation results and experiments has been attributed to the interaction mechanism operating among the dust particles [54]. The effect of ion flow is not reflected when simulation is performed with isotropic Yukawa potential.

The present study reveals that the correct treatment of interaction potential in presence of magnetized wake opens avenues for new magnetic field dependent shear viscosity. A new regime of parameters has been identified with Γ = 450, $\kappa = 2$, $\nu = 1$ Hz, $B = 0.001 - 0.09$ T in the super sonic ion flow regime, where shear viscosity shows sharp variation with small change in the magnetic field. In the Yukawa dominant, strongly coupled liquid state, the viscosity is found to vary as B^7 with magnetic field in the range 0.001 T - 0.05 T and B^4 in the range 0.05 T - 0.09 T. On the other hand, when the system moves towards a gaseous state, the viscosity is found to vary as $\frac{1}{B^2}$ with magnetic field in the range 0.09 T - 0.3 T. In absence of magnetized wake potential, such response of shear viscosity to low magnetic field is not manifested. Although Yukawa potential does not depend on magnetic field, the tunable wake potential controls the effective interaction among the dust grains and plays a crucial role in bringing out the unique nature of shear viscosity. We have also seen that for moderate values of Mach number, in the gas-like state, the viscosity does not change much whereas for strongly correlated fluid state characterized by high Mach number ($M \geq 1.5$), the viscosity shows significant rise with M. It is to be noted that the viscosity of such system is closely connected with the phase state of dusty plasma.

Feng et al. [48], in their work observed that when a magnetic field is increased, the shear viscosity of a 2D liquid dusty plasma increases at low temperatures, while at high temperatures its viscosity diminishes. This behaviour may be explained on the basis of phase state of the system. In low temperatures, our results are in agreement with that of Feng et al., in absence of wake potential. However, the behaviour of shear viscosity in low magnetic field regime as discussed above does not manifest itself in absence of magnetized wake.

The dust neutral collision frequency may have important role on the rheological property of the medium. In absence of wake potential, we observe a rise in shear viscosity with dust neutral collision frequency of the medium and the results show similar trend with the experimental results reported by Gavrikov et al. [54]. It would be interesting to see the effect of neutral pressure and dust neutral collision on the rheological behaviour of the medium. It is found that the wake potential gets destroyed in presence of strong ion neutral collision frequency and therefore a completely different model may be required to see the dependence of shear viscosity on neutral pressure [55].

Generalized hydrodynamic model is widely used to study collective modes in strongly coupled liquid dusty plasma where the dust particles are treated as viscoelastic fluid. Correct estimate of shear viscosity of such medium may have important consequence on the behaviour of collective modes and nonlinear structures. Shear viscosity may have an important role to play on the onset of vortex in dusty plasma. In a recent experiment, Bailung et al. [56], have reported that a pair of vortex is formed when Reynold number falls in the range 60-90, where they have ignored the dependence of shear viscosity on particle interaction completely. The present article shows the way how to calculate shear viscosity in laboratory condition where ion flow induced wake may be dominant and hence predict correct range of Reynold number for which vortex structures may appear in dusty plasma.

Due to the unique property of tunable viscosity, complex plasma may be used as a platform to study magneto-rheological characteristics of soft matter and there is a possibility of using dusty plasma as a magneto-rheological material in near future.

CRediT authorship contribution statement

Biswajit Dutta: Formal analysis, Investigation, Methodology, Writing – original draft. **Hirakjyoti Sarma:** Methodology. **Pratikshya Bezbaruah:** Writing – review & editing. **Nilakshi Das:** Writing – original draft, Supervision, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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